647510 **BSC2MatC201x** **Seat No :________**

B.Sc. Semester - 2 (*CBCS*) Examination

June/July-2022 [NEW COURSE]

MATHEMATICS (CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q-1 (A) Answer any Two out of Three. **[10]**

(1) Find the equation of tangent plane to the sphere with (2,-1,4) and (-2,2,-2) as extremities of diameter at point (2,-1,4) . Also find equation of normal to this sphere.

(2) Find equation of right circular cylinder with axis $\{(3,4,5+k) / k\epsilon R\}$ and radius 2.

(3) Find center and radius of the circle $x^2+y^2+z^2-2y-4z-20=0$; $x+2y+2z=15$.

(B) Answer any One out of Two **[4]**

(1) Show that two circles $x^2+y^2+z^2-y+2z=0$; $x-y+z-2=0$ and $x^2+y^2+z^2+x-3y+z-5=0$; $2x-y+4z-1=0$ lie on the same sphere and find of equation of it.

(2) Find the equation of the sphere passing through the points (0,0,0), (-a,b,c),(a,-b,c),(a,b,-c).

Q-2(A) Answer any Two out of Three. **[10]**

(1) If $u=\log(x^2+y^2)+\tan^{-1}(y/x)$, find the value of $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}$.

(2) If $x^3+y^3+z^3-3xyz=0$ then find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

(3) If $u=\tan^{-1}\frac{x^3+y^3}{x-y}$ then show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$.

(B) Answer any One out of Two **[4]**

(1) Find $\lim\limits_{(x,y)\to(0,0)}\frac{e^{x+y}-e^x-e^y+1}{xy}$.

(2) $f(x,y)=\frac{x^2-xy}{x+y}$;$(x,y)\neq(0,0)$ and $f(0,0)=0$, then find $f_x(0,0)$ and $f_y(0,0)$.

Q-3 (A) Answer any Two out of Three. **[10]**

(1) Expand $e^x \cos y$ in powers of x and y up to three degree.

(2) If $u=x+y+z$, $uv=y+z$, $uvw=z$ then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.

(3) State and prove Taylor's theorem.

(B) Answer any One out of Two **[4]**

(1) If $f(x,y)=x^2y-3y$ then find the approximate value of f(5.12,6.85)

(2) In usual notation prove that $\frac{\partial(x,y,z)}{\partial(r,\theta,\varnothing)}=r^2\sin\varnothing$.

Q-4 (A) Answer any Two out of Three. **[10]**

(1) In usual notation prove that $(A+B)^T=A^T+B^T$ and $(AB)^T=B^T A^T$

(2) Prove that every square Matrix can be expressed uniquely as the sum of a symmetric and a skew symmetric Matrices.

(3) If $A=\begin{bmatrix}0 & 2m & n\\ l & m & -n\\ l & -m & n\end{bmatrix}$ *is* orthogonal then find values of l,m,n.

(B) Answer any One out of Two **[4]**

(1) If A=$\begin{bmatrix} cos\theta & \sin\theta \\ -\sin\theta & cos\theta \end{bmatrix}$ then prove that A^n=$\begin{bmatrix} cosn\theta & \sin n\theta \\ -sinn\theta & cosn\theta \end{bmatrix}$.

(2) Find rank of matrix $\begin{bmatrix} 3 & 1 & 4 & 0 \\ 0 & 2 & 2 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix}$ using echelon form.

Q-5 (A) Answer any Two out of Three. **[10]**

(1) If A=$\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ then express $2A^5$-$3A^4$+A^2-4I as a linear polynomial.

(2) Varify Cayley- Hamilton theorem for $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 2 & 3 & 0 \end{bmatrix}$.

(3) Find eigen values and eigen vectors of Matrix A= $\begin{bmatrix} 5 & 3 \\ -1 & 1 \end{bmatrix}$.

(B) Answer any One out of Two **[4]**

(1) Solve the equations 5x+2y+2z=17; 9x+4y+2z=27.

(2) Solve the equations x+3y+2z=0; x+4y+3z=0; x+5y+4z=0.

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